§29. The Flierl-Petviashvili Equation and the Zonal Flow

Spineanu, F., Vlad, M. (Association EURATOM-MEC, NILPRP, Romania)
Itoh, K., Sanuki, H. (NIFS, Toki)
Itoh, S.-I. (RIAM, Kyushu Univ.)

The drift instability in tokamak plasma exhibits distinct types of structures on different spacial scales. A multiple space-time scale analysis of the governing equations (continuity, ion momentum and adiabaticity) has revealed that on scales intermediate between the large linear cells and the sonic Larmor radius (ρ_s) the stationary states are solutions of the Flierl-Petviashvili equation

$$\Delta\phi = \alpha\phi - \beta\phi^2 \qquad (1)$$

$\alpha = (1 - v_*/u)/\rho_s^2$, $\beta = (c_s^2/2u^2)\, d\left(L_n^{-1}\right)/dx$. Starting from the one-dimensional solution $\phi(y) = 3\alpha/(2\beta)\cosh^{-2}\left(\sqrt{\alpha}y/2\right)$ we look for the dependence of the complex-y singularities on the other coordinate, x. We find that the solution is a doubly periodic meromorphic function

$$\phi_s(x, y) = \frac{\alpha}{2\beta} + s\wp\left(iay + ibx + \omega | g_2, g_3\right) \qquad (2)$$

$g_2 = 3\alpha^2/(s\beta)^2$ and g_3 are the invariants of the Weierstrass elliptic functions, $\wp$. Eq.(2) is the exact, periodic, solution of the Flierl-Petviashvili equation. It has a one dimensional geometry consisting of layers of periodic flow (Fig.1) with an orientation in the (y, x) plane given by a/b which verifiy $a^2 + b^2 = s\beta/6$. g_3 and s are a constants of integration fixed by the physical conditions when the system evolves to this solution. When $g_3 = 0$ the width of the layer of poloidal flow is $\delta x = 3.924K(k')(1 - v_*/u)^{-1/2}\rho_s$.

The pattern is the same as the zonal flow in plasma. Experimental studies on Doublet III-D tokamak have obtained $\widetilde{\varphi}_{rms} > 10\,V$, $\lambda_r \in (15...30)\,\rho_s$, and a flow shear $\omega_{E\times B} \sim 2 \times 10^5\,s^{-1}$. From Eq.(2) we obtain: $\widetilde{\varphi}_{rms} > 17\,V$, $\lambda_r \simeq 17.4\rho_s$, $\omega_{E\times B} \sim 2.2 \times 10^5\,s^{-1}$. Results of gyrokinetic simulations at Lausanne have obtained $\delta x \sim [8...13]\,\rho_s$, $E_r \in [-24, +24] \times 10^3\,V/m$ and the flow shear rate $\omega_{E\times B} \in [-5, +5] \times 10^{-3}\Omega_i$. We obtain $\delta x \simeq 13\rho_s$, $E_r \in [-23, +23] \times 10^3\,V/m$ and $\omega_{E\times B} \in [-10, +10] \times 10^{-3}\Omega_i$.

In the presence of a perturbation the flow evolves in time. The scalar nonlinearity equation can be written in the form ($4\eta^2$ is a constant)

$$\frac{\partial}{\partial t}\left(1 - \nabla_\perp^2\right)\phi = \frac{\partial}{\partial y}\left[\left(-\nabla_\perp^2 + 4\eta^2\right)\phi\right] - \phi\frac{\partial\phi}{\partial y}$$

We obtain the linear dispersion relation

$$\frac{1}{k_y} = \frac{\omega \pm Q^{1/2}}{P+1}\left\{1 \mp \left[1 + \frac{2(P+1)}{\left(-\omega + Q^{1/2}\right)^2}\right]^{1/2}\right\}$$

where $P \equiv -4\eta^2 + \left(\phi_s|_{\min} + \phi_s|_{\max}\right)/2$ and $Q \equiv (\delta x)^2\left(\phi_s|_{\max} - \phi_s|_{\min}\right)^2 \equiv (\delta x)^2(\delta\phi_s)^2$. The vortical structures have $\lambda_y \sim 5\rho_s$.

Extensive numerical simulations have been done for studying the stability and the structural stability (*i.e.* including both the scalar and the vectorial nonlinearities) of ϕ_s. For small monopolar perturbations the total flow is stable for more than $12 \times 10^3\Omega_i$. The monopole is reshaped at early stages for equality of the tangential flows and then is stably advected ($v_*/u \lesssim 1$). This result may be relevant for the stability of the Red Spot, the long-lived vortex embedded in the zonal flows of the atmosphere of Jupiter, a geometry very similar to ours. A small amplitude dipolar initial perturbation is also stable for similar durations.

This exact solution has the necessary characteristics to represent the zonal flow as a coherent nonlinear structure and strongly supports the role of the scalar nonlinearity in the phenomenology of the zonal flows [1].

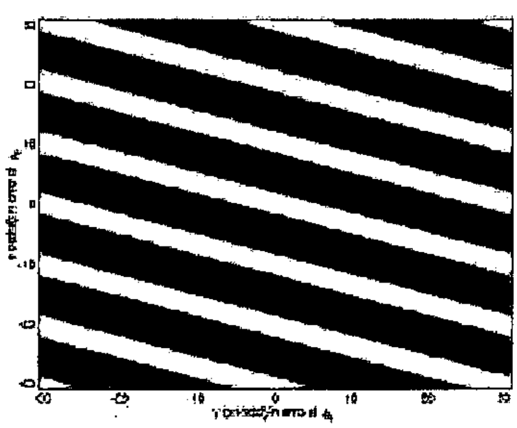

Figure 1: Amplitude of the perturbed potential from the solution ϕ_s for $a/b = 0.9$

References

[1] F. Spineanu, M. Vlad, K. Itoh, H. Sanuki and S.-I. Itoh, Phys. Rev. Letters, (2004) in print.